Journal of Engineering Physics and Thermophysics, Vol. 77, No. 1, 2004

RESONANCE OSCILLATIONS OF A GAS IN AN OPEN-END PIPE IN THE TURBULENT REGIME

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UDC 534.213;536.244;532.517.4

Consideration has been given to the theory of oscillating flows in a pipe for the case where turbulence develops monotonically, reaching the pipe axis within a certain period of time after the beginning of acceleration. A mathematical model describing resonance oscillations of this type in an open pipe and consistent with experiment has been constructed.

It is common knowledge that resonance oscillations are set up in a pipe at one end of which there is a harmonically oscillating piston and the other end of which communicates with the ambient medium. These oscillations are accompanied by the formation of a pulsating jet, turbulization of the flow, and a number of other nonlinear effects [1–4]. Interest in such systems is maintained owing to their wide acceptance in technology.

Nonresonance oscillating turbulent flows have been investigated in a number of works [5–8]. In some works [5, 6], the logarithmic velocity profile is observed throughout the oscillation period and it occupies the entire cross section of the pipe. At any instant, hydraulic resistance obeys the Blasius law. In others [7, 8], it has been found that the thickness of the logarithmic layer monotonically increases with time until the layer occupies the entire cross section of the pipe. Until the logarithmic layer reaches the channel axis, the velocity maximum is observed at a certain distance from the wall and it is much larger than the velocity on the tube axis at this instant of time. According to [9], the behavior of the flows is determined by the speed of growth of the logarithmic layer. If the turbulence reaches the pipe at the early stages of acceleration, one observes flows of the type of [5, 6]; if, conversely, the turbulence propagates slowly, the cases of [7, 8] are realized.

To calculate resonance oscillations one must prescribe the dependences of the tangential stress on the wall on the velocity oscillations and of the heat-flux density on the pressure oscillations, which are nonlinear in turbulent flows. An approach based on linearization of the above quantities by one method or another has been proposed in [10] with the aim of overcoming this circumstance. In particular, one can prescribe in advance the dependence of the velocity oscillations on the axial coordinate and average it over the pipe length.

Resonance oscillations in the case where turbulence reaches the pipe axis at the early stages of acceleration and a logarithmic velocity profile (flows of the type of [5, 6]) is rapidly established on the entire cross section of the pipe have been considered in one of the first isentropic models, where the tangential stress on the wall was assumed to obey the Blasius law [11]. The linearized expression of tangential stress was substituted into the acoustic equations.

Resonance oscillations in the regime of the so-called weakly developed turbulence where the profile of the amplitude of velocity oscillations is assumed to be uniform everywhere except for a thin logarithmic layer in the vicinity of the wall (the logarithmic layer develops so slowly that the turbulence has no time to propagate to the pipe axis) have been considered in [12].

In the present work, we seek to construct a model of resonance oscillations in the case of slow propagation of turbulence; this case corresponds to flows of the type of [7, 8].

Resonance oscillations in a narrow cylindrical pipe of length L and radius R ($R << L$) at one end of which there is a harmonically oscillating piston with a small displacement amplitude $l_0 << L$ and the other end of which communicates with the ambient medium are characterized by the set of dimensionless parameters [12]

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 77, No. 1, pp. 109–113, January–February, 2004. Original article submitted February 18, 2003.

$$\varepsilon = \frac{V}{\omega L} , \quad H = R \sqrt{\frac{\omega}{\nu}} , \quad \mathrm{Sh} = \frac{\omega R}{V} , \quad \mathrm{M_{pstn}} = \frac{\omega l_0}{c_0} , \quad \mathrm{Re} = \frac{V^2}{\omega \nu} . \tag{1}$$

In oscillations at the fundamental frequency, the condition $l_0 << L$ is equivalent to $\mathrm{M_{pstn}} << 1$. Of principal practical interest is the case $H >> 1$. Let $\mathrm{Sh} << 1$; then we have $\varepsilon << 1$ for $R/L << 1$, i.e., the problem can be solved by the perturbation method. Finally, the flow will be turbulent if $\mathrm{Re} \geq 1.6 \cdot 10^5$ [7].

The equations of the first (acoustic) approximation can be represented in the form

$$\rho_0 \frac{\partial u_{1s}}{\partial t} + \frac{\partial p_1}{\partial x} = - \frac{2\tau_1}{R} ,$$

$$\frac{\partial p_1}{\partial t} + \rho_0 c_0^2 \frac{\partial u_{1s}}{\partial x} = \frac{2 (\kappa - 1) q_1}{R} . \tag{2}$$

The relationship between the amplitude of tangential stress on the wall and the amplitude of velocity oscillations in the case of a uniform velocity distribution over the pipe length is as follows [7]:

$$\tau_1 = \frac{1}{2} \rho_0 f_w u_{1m}^2 . \tag{3}$$

Representation of experimental data [7] in analytical form leads to the expression

$$f_w = 0.066 (\omega \nu)^{0.2} u_{1m}^{-0.4} . \tag{4}$$

In resonance oscillations, u_{1m} is a function of the axial coordinate; therefore, dependence (3) must be linearized. For this purpose we write τ_1 in the form

$$\tau_1 (x, t) = \rho_0 \beta_0 u_1 (x, t + \varphi) , \tag{5}$$

where

$$\beta_0 = \frac{1}{L} \int_0^L f_w u_{1m} (x) \, dx . \tag{6}$$

We take the leading, term of the amplitude of velocity oscillations to be expressed in the form $u_{1m}(x) = V \sin k_0 x$, where $k_0 = \omega / c_0$. Then with account for $k_0 L \approx \pi / 2$ we obtain

$$\beta_0 = 0.024 (\omega \nu)^{0.2} V^{0.6} . \tag{7}$$

Experience shows [7] that tangential stress on he wall leads the velocity oscillations by the angle φ_0; consequently, (5) can be represented as

$$\tau_1 (x, t) = \rho_0 \beta_0 u_1 (x, t) \exp (i \varphi_0) \tag{8}$$

or, with account for $u_{1s}(x, t) = B u_1(x, t) \exp (i \varphi_1)$, in the form

$$\tau_1 = \rho_0 \beta u_{1s} \exp (i \varphi) , \quad \beta = \frac{\beta_0}{B} , \quad \varphi = \varphi_0 - \varphi_1 . \tag{9}$$

For evaluation of q_1 we consider the relation

$$\frac{q_1}{\tau_1} = - \frac{(\lambda + \lambda_t)}{(\mu + \mu_t)} \frac{(\partial T_1 / \partial r)_{r=R}}{(\partial u_1 / \partial r)_{r=R}} . \tag{10}$$

Let Pr = 1; then $\lambda = c_p\mu$ and $\lambda_t = c_p\mu_t$ and the dimensionless fields of temperatures and velocities are similar, i.e.,

$$\left(\frac{\partial \theta_1}{\partial \xi}\right)_{\xi=1} = \left(\frac{\partial \bar{u}_1}{\partial \xi}\right)_{\xi=1} , \tag{11}$$

where $\theta_1 = T_1/T_{1m}$, $\bar{u}_1 = u_1/u_{1m}$, and $\xi = r/R$.

In the flow core, we have $p_1 = \rho_0 c_p T_{1m}$; then, with account for (11), from (10) we easily obtain

$$q_1 = -\beta_T p_1 , \quad \beta_T = \beta_0 \exp(i\varphi_0) . \tag{12}$$

We substitute (9) and (12) into (2) and pass to dimensionless variables

$$\frac{1}{c_0}\frac{\partial \bar{u}_{1s}}{\partial t} + \frac{\partial \bar{p}_1}{\partial x} = -a_1^* \bar{u}_{1s} ,$$

$$\frac{1}{c_0}\frac{\partial \bar{p}_1}{\partial t} + \frac{\partial \bar{u}_{1s}}{\partial x} = -a_2^* \bar{p}_1 , \tag{13}$$

where

$$a_1^* = (2\beta_0/BRc_0)\exp(i\varphi) ; \quad a_2^* = (2(\kappa-1)\beta_0/Rc_0)\exp(i\varphi_0) ;$$

$$\bar{p}_1 = p_1/\rho_0 c_0^2 ; \quad \bar{u}_{1s} = u_{1s}/c_0 .$$

We set $\bar{p}_1(x, t) = \bar{p}_1(x)\exp i(\omega t + \psi_1)$ and $\bar{u}_{1s}(x, t) = \bar{u}_{1s}(x)\exp i(\omega t + \psi_1)$ in (13) and eliminate one variable, for example, $\bar{u}_{1s}(x)$. Then for the pressure-oscillation amplitude we have

$$\frac{\partial^2 \bar{p}_1(x)}{\partial x^2} - \left(\frac{i\omega}{c_0} + a_1^*\right)\left(\frac{i\omega}{c_0} + a_2^*\right)\bar{p}_1(x) = 0 . \tag{14}$$

If, as the solution of (14), we select

$$\bar{p}_1(x) = r_1 \cos z_1 , \tag{15}$$

where $z_1 = k_1 x + \alpha_1 + i\beta_1$, after substituting it into (14) we obtain the dispersion relation

$$k_1^2 = \left(\frac{i\omega}{c_0} + a_1^*\right)\left(\frac{i\omega}{c_0} + a_2^*\right). \tag{16}$$

Substituting the quantities a_1^* and a_2^* into (16) and assuming that $a_1^*/k_0 << 1$ and $a_2^*/k_0 << 1$, where $k_0 = \omega/c_0$, with a sufficient degree of accuracy, we have

$$k_1 = k_0 + b_1 + ib_2 , \tag{17a}$$

$$b_1 = \beta_0 \frac{\sin\varphi + (\kappa-1)B\sin\varphi_0}{BRc_0} , \quad b_2 = \beta_0 \frac{\cos\varphi + (\kappa-1)B\cos\varphi_0}{BRc_0} . \tag{17b}$$

The amplitude of velocity oscillations $\bar{u}_{1s}(x)$ is determined from the formula

$$\bar{u}_{1s}(x) = -\frac{ik_1 r_1}{k_0 - ia_1^*}\sin z_1 , \tag{18}$$

then the solution of system (13) can be written in the form

$$\bar{p}_1 (x, t) = r_1 \cos z_1 \exp i (\omega t + \psi_1) ,$$

$$\bar{u}_{1s} (x, t) = - i r_1 B \sin z_1 \exp i (\omega t + \psi_1 + \varphi_1) , \tag{19}$$

where

$$B = \left| k_1 / (k_0 - i a_1^*) \right| ; \ \varphi_1 = \arg (k_1 / (k_0 - i a_1^*)) .$$

With a sufficient degree of accuracy we have

$$B \cong 1 , \quad \varphi_1 \cong 0 , \tag{20}$$

i.e., the velocity average over the cross section differs little from its maximum value $\bar{u}_{1s} \cong \bar{u}_1$.

With account for (20) expressions (17b) take the form

$$b_1 = \frac{\beta_0 \kappa \sin \varphi_0}{R c_0} , \quad b_2 = \frac{\beta_0 \kappa \cos \varphi_0}{B R c_0} . \tag{21}$$

We consider the boundary conditions. At the end closed by the piston, we prescribe the piston velocity

$$\bar{u}_1 (0, t) = M_{\text{pstn}} \exp i (\omega t) . \tag{22}$$

The procedure of calculation of the boundary condition at the open end, which is based on the idea of the jet character of outflow and spherical flow into the sink in the outlet cross section of the pipe, has been given in [12] for the case of harmonic velocity oscillations. We assume that at a certain distance from the outlet cross section inside the pipe the velocity varies according to the law

$$\bar{u}_1 (L, t) = \bar{v} \sin (\omega t + \psi_1) , \quad \bar{v} = V / c_0 . \tag{23}$$

Then the oscillations at the fundamental frequency can be written in the form

$$\bar{p}_1 (L, t) = m \bar{v}^2 \sin (\omega t + \psi_1) ,$$

$$m = 0.5 \left\{ 2 (0.5 m_0 + a_0) (0.5 + a_1) - (0.5 + a_1) a_2 + a_2 a_3 - a_3 a_4 + a_4 a_5 \right\} , \tag{24}$$

where a_i are the coefficients of Fourier-series expansion of the jet velocity at the distance $x \cong 3R$ from the outlet cross section of the pipe [12] and m_0 is the coefficient determined from the equation

$$3 m_0 \pi - 2 (m_0 \arcsin m_0 + \sin \arccos m_0) = 0 . \tag{25}$$

Substituting (19) into (22) and (24), for determination of r_1, ψ_1, α_1, and β_1 we obtain the system of equations

$$r_1 \sin \alpha_1 \cosh \beta_1 = M_{\text{pstn}} \sin \psi_1 , \quad r_1 \cos \alpha_1 \sinh \beta_1 = M_{\text{pstn}} \cos \psi_1 ,$$

$$\cos z \cosh w = m r_1 \sqrt{\sin^2 z + \sinh^2 w} \cos z \sinh w ;$$

$$\sin z \sinh w = m r_1 \sqrt{\sin^2 z + \sinh^2 w} \sin z \cosh w , \tag{26}$$

where $z = (k_0 + b_1) L + \alpha_1$ and $w = \beta_1 - b_2 L$.

System (26) for $r_1 << 1$, $\sinh w \sim r_1$, and $\cosh w \approx 1$ is easily solved in the following manner:

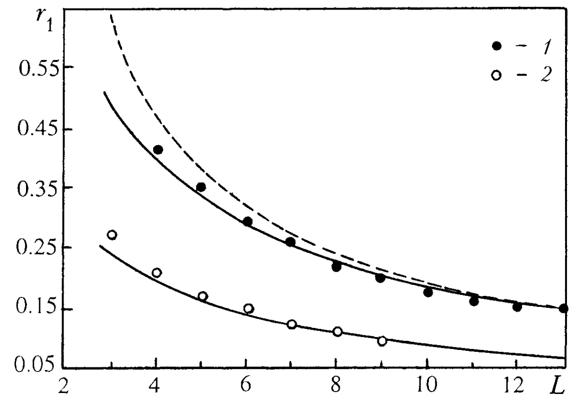

Fig. 1. Dimensionless oscillation amplitudes vs. pipe length: 1) m_1 = 2.2727 and 2) 1.1818. L, m.

$$\alpha_1 = \frac{\pi}{2} - (k_0 + b_1)\, L\,, \quad \beta_1 = mr_1 + b_2 L\,, \quad \psi_1 = \arctan\,(\tan \alpha_1 \cot \beta_1)\,,$$

$$r_1 \sqrt{\cos^2 (k_0 + b_1)\, L + (mr_1 + b_2 L)^2 \sin^2 (k_0 + b_1)\, L} = M_{\text{pstn}}\,, \tag{27}$$

where b_1 and b_2 also depend on r_1.

At sharp resonance ($\alpha_1 = 0$), we have

$$(k_0 + b_1)\, L = \frac{\pi}{2}\,, \quad \beta_1 = mr_1 + b_2 L\,,$$

$$r_1\,(mr_1 + b_2 L) = M_{\text{pstn}}\,, \quad \psi_1 = 0\,. \tag{28}$$

It is easy to show that in the case where the turbulent boundary layer reaches the axis at the early stages of acceleration and the coefficient of friction is determined by the Blasius law, β_0 is independent of the oscillation frequency. In our case $\beta_0 \sim \omega^{0.2}$. The dependence of β_0 on ν and V is nearly the same in both cases.

The points in Fig. 1 show the experimental data [13] obtained in a pipe with a tapered reducer for two values of $m_1 = d_{\text{pstn}}/d_{\text{pp}}$ when l_0 = 0.04575 m (the solid curves denote results of the calculation from formula (27)). The effective amplitude of displacement of the piston $l_{\text{ef}} = m_1^2 l_0$ has been employed for theoretical calculations [14]. The dashed curve in the figure is calculation from the formula $r_1 = 1.9084/L$ for the pipe with m_1 = 2.2727. Satisfactory agreement of the data is seen. Noteworthy is a monotone decrease in the dimensionless amplitude of oscillations with increase in the pipe length. The dependence of r_1 on the pipe length, calculated from (27) (solid curve for m_1 = 2.2727), is quite similar to the inversely proportional dependence (dashed curve), particularly for longer pipes, as indicated by Repin et al. [13].

Thus, the model proposed is suitable for description of resonance oscillations in turbulent flows in the cases where turbulence reached the pipe axis within a certain time after the beginning of acceleration.

NOTATION

a_i, coefficients of the Fourier series; a_1^* and a_2^*, coefficients of linearization of the tangential stress on the wall and of the heat flux, m^{-1}; B, dimensionless parameter allowing for the displacement of the flow by the boundary layer; b_1 and b_2, dispersion and absorption coefficients determined by turbulent friction and heat conduction, m^{-1}; c_0, velocity of sound in the unperturbed gas, m/sec; c_p, specific heat at constant pressure, J/(kg·K); d_{pstn}, piston diameter, m; d_{pp}, pipe diameter, m; f_{w}, coefficient of friction on the wall; H, frequency parameter; k_0, wave number of the ideal gas, m^{-1}; k_1, resultant wave number in the turbulent flow, m^{-1}; L, pipe length, m; l_0, amplitude of displacement of the piston, m; l_{ef}, effective amplitude of displacement of the piston, m; m, factor of proportionality between velocity and pressure oscillations at the open end of the pipe; m_0, proportionality factor obtained from the law of conservation of mass at the open end of the pipe; m_1, coefficient allowing for the geometric parameters of the tapered reducer; M_{pstn}, Mach number for the piston; p, pressure, Pa; Pr, Prandtl number; q, heat flux, W/m^2; r, radial coordinate, m; R, pipe radius, m; r_1, modulus of the dimensionless oscillation amplitude; Re, Reynolds number; Sh, Strouhal number; t, time, sec; T, temperature, K; u, velocity, m/sec; u_{1s}, effective velocity, m/sec; $\bar{v}$, dimensionless amplitude of velocity oscillations in the vicinity of the open end of the pipe; V, amplitude of velocity oscillations in the vicinity of the open end of the pipe, m/sec; x, axial coordinate, m; z_1, argument of the distribution function of the pressure and velocity amplitude over the pipe length; z and w, real and imaginary parts of the function z_1 at the open end of the pipe; α_1 and β_1, integration constants; β_0, coefficient of linearization of the tangential stress over the pipe length, m/sec; β_t, coefficient allowing for the relationship between the pressure and heat-flux oscillations, m/sec; ε, nonlinearity parameter; θ, dimensionless temperature; κ, Karman constant; λ, thermal conductivity, W/(m·K); λ_t, turbulent thermal conductivity, W/(m·K); μ, coefficient of dynamic viscosity, kg/(m·sec); μ_t, coefficient of turbulent viscosity, kg/(m·sec); ν, coefficient of kinematic viscosity, m^2/sec; ξ, dimensionless radial coordinate; ρ_0, density of the unperturbed gas, kg/m^3; τ_1, tangential stress on the wall, N/m^2; φ_0, phase shift between the tangential stress on the wall and the velocity oscillations; φ_1, principal value of the argument of the function allowing for the displacement of the flow by the boundary layer; φ, difference of the phases φ_0 and φ_1; ψ_1, principal value of the argument of the dimensionless amplitude of oscillations; ω, cyclic frequency, 1/sec. Subscripts and superscripts: 1, first (acoustic) approximation; $\bar{\ }$, dimensionless quantity; m, oscillation amplitude; s, averaging of the quantity over the pipe cross section; t, turbulent; pstn, piston; w, wall; ef, effective; pp, pipe.

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